Mathematics: analysis and approaches
Standard Level
Paper 1

Name

Date: _____

1 hour 30 minutes

_____

Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

exam: 9 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Consider the right square pyramid shown below. Given that the area of the square base is 36 cm^2 and the volume of the pyramid is $36\sqrt{3}$ cm^3, find the angle θ between the base of the pyramid and one of its lateral faces.
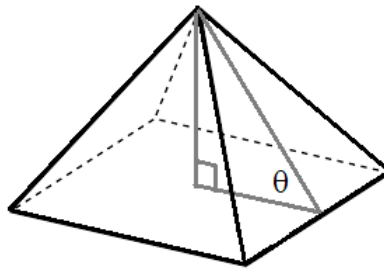
2. [Maximum mark: 5]

Let A and B be events such that $P(A \cap B) = \dfrac{1}{5}$, $P(B \mid A) = \dfrac{1}{2}$ and $P(A \mid B) = \dfrac{3}{10}$.

Find $P(A \cup B)$.

3. [Maximum mark: 4]

(a) A two-digit number n is written in the form $10a+b$, where a and b are integers. The two-digit number m is formed by reversing the digits of n. Express m in terms of a and b. [1]

(b) Hence, or otherwise, prove that the difference between a two-digit number and its reverse is a multiple of nine. [3]

4. [Maximum mark: 6]

Let $h'(x) = x\sqrt{1-x^2}$. Given that $h(0) = \dfrac{2}{3}$, find $h(x)$.
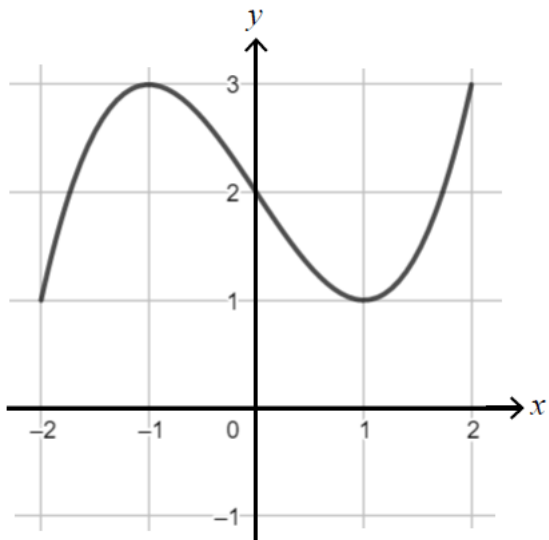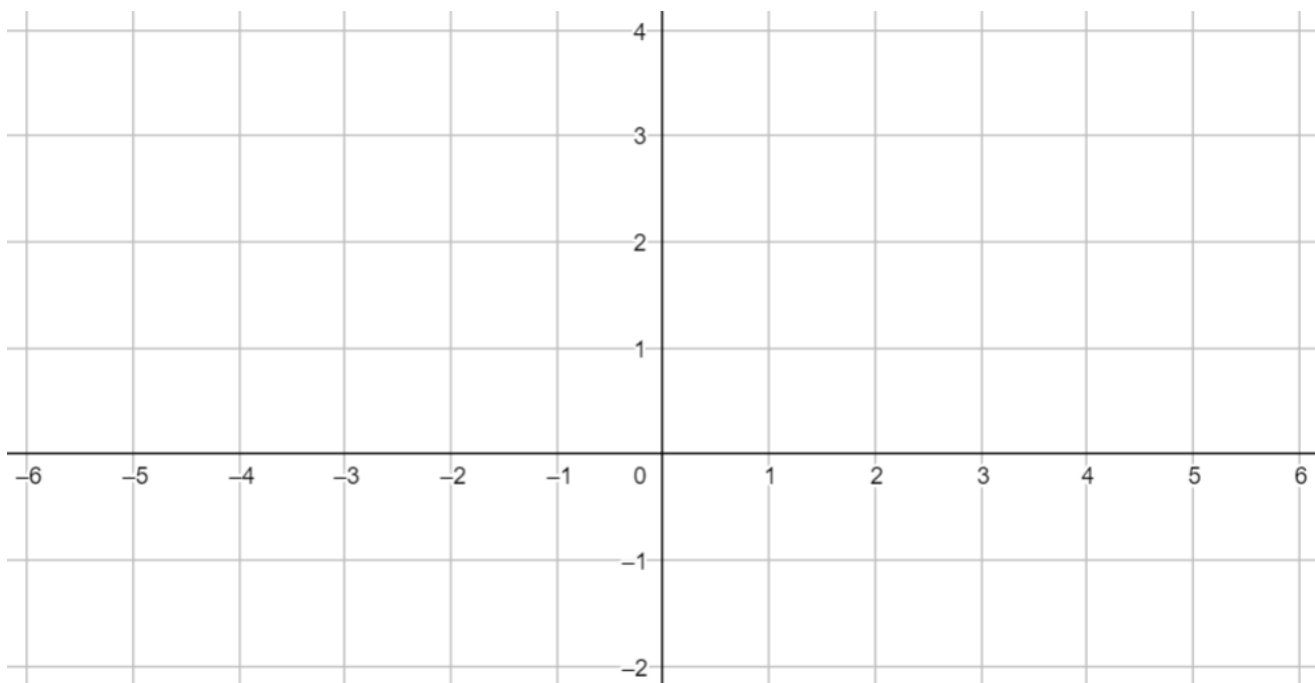
5. [Maximum mark: 5]

The following diagram shows the graph of $y = f(x), -2 \le x \le 2$. The graph has a horizontal tangent at the points $(-1, 3)$ and $(1, 1)$.

On the set of axes below, sketch the graph of $y = f\left[2(x-2)\right]$, clearly indicating the coordinates of any local maxima or minima.

6. [Maximum mark: 8]

Solve for x in each equation.

(a) $\ln x + \ln(x-2) - \ln(x+4) = 0$ [4]

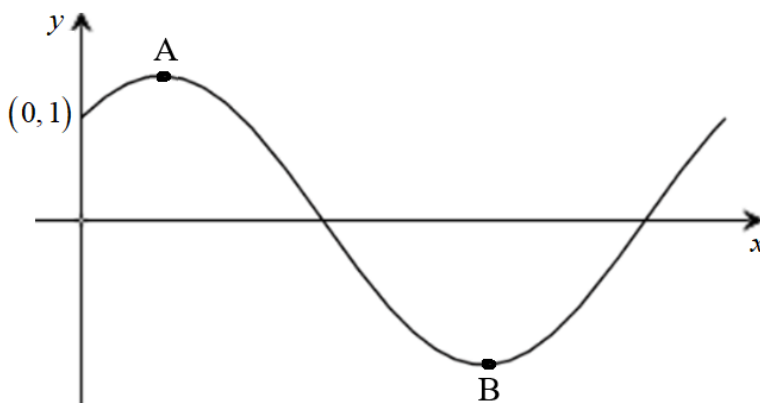(b) $\log_3(4x^2 - 5x - 6) = 1 + 2\log_3 x$ [4]

Do **not** write solutions on this page.

Section B

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

7. [Maximum mark: 17]

Consider the function f (graph shown below) defined by $f(x) = \cos x + \sin x, \ 0 \le x \le 2\pi$.

The *y*-intercept is at $(0,1)$, there is a maximum point at $A(p, q)$ and a minimum point at B.

(a) Find $f'(x)$.   [2]

(b) Hence

    (i) show that $q = \sqrt{2}$;

    (ii) verify that A is a maximum.   [10]

(c) State the coordinates of B.   [3]

The function $f(x)$ can be written in the form $r\cos(x-c)$ where $r, c \in \mathbb{R}$.

(d) Write down the value of r and the value of c.   [2]

Do **not** write solutions on this page.

8.   [Maximum mark: 12]

A bag A contains 2 red balls and 3 yellow balls. A bag B contains 4 red balls and 2 yellow balls. Two balls are randomly selected from one of the bags. If bag A is chosen, then the probabilities are as follows:

$$P(2 \text{ red balls}) = \frac{1}{10}$$

$$P(2 \text{ yellow balls}) = \frac{3}{10}$$

$$P(1 \text{ red ball and 1 yellow ball}) = \frac{6}{10}$$

(a) Calculate the probabilities for the same three outcomes if bag B is chosen.   [5]

In order to decide which bag to choose, a fair die with six faces is rolled. If a 1 or 6 is rolled, then bag A is chosen. If a 2, 3, 4 or 5 is rolled, then bag B is chosen.

The die is rolled and then two balls are drawn from the selected bag.

(b) Calculate the probability that two red balls are selected.   [3]

(c) Given that two red balls are obtained, find the probability that a 1 or 6 was rolled on the die.   [4]

9.   [Maximum mark: 18]

The function g is defined by $g(x) = \dfrac{x}{e^{x^2}}$, where $x \geq 0$.

(a) Show that there is one maximum point P on the graph of g, and find the x-coordinate of P. [5]

(b) Show that g has a point of inflexion Q at $x = \sqrt{\dfrac{3}{2}}$.   [6]

(c) Determine the intervals on the domain of g where g is

    (i)   concave up

    (ii) concave down.   [2]

(d) The region bounded by the graph of g, the x-axis and the vertical line $x = k$ has an area equal to $\dfrac{1}{2} - \dfrac{1}{2e^4}$. Find the value of k.   [5]